Validating the Earth's Core using Atmospheric Neutrinos with ICAL at INO Based on JHEP 08 (2021) 139, arXiv:2104.11740

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### A Brief Review of the Internal Structure of Earth

The current understanding about the Earth is from Seismic studies.

- Crust: solid, rocks, brittle, lowest density
- Mantle: hot, solid upper mantle, viscous plastic lower mantle
- Core: solid inner core, liquid outer core, iron and nickel

Region	R <sub>min</sub> (km)	R <sub>max</sub> (km)	Density (g/cm <sup>3</sup> )	
Core	0	3480	11.37	
Mantle	3480	5701	5	
Crust	5701	6371	3.3	



#### References:

E. C. Robertson, The interior of the Earth, an elementary description, 1966.

D. E. Loper and T. Lay, The core-mantle boundary region, Journal of Geophysical Research: Solid Earth 100 (1995), no. B4 6397–6420.

D. Alfe, M. J. Gillan, and G. D. Price, Temperature and composition of the earth's core, Contemporary Physics 48 (2007), no. 2 63–80.

Anil Kumar, Sanjib Kumar Agarwalla, JHEP 08 (2021) 139, arXiv: 2104.11740 👘 🗐 🗐 🖉 🧟 🖓 🔍 🔿

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### Atmospheric Neutrinos



$$\begin{split} \pi^+ &\rightarrow \mu^+ + \nu_\mu \\ \mu^+ &\rightarrow e^+ + \nu_e + \bar{\nu}_\mu \\ \pi^- &\rightarrow \mu^- + \bar{\nu}_\mu \\ \mu^- &\rightarrow e^- + \bar{\nu}_e + \nu_\mu \end{split}$$

 $\begin{array}{l} \mbox{Expectation:} \ \frac{\nu_{\mu}+\bar{\nu}_{\mu}}{\nu_{e}+\bar{\nu}_{e}}\sim 2 \\ \mbox{but at high energies } \ \frac{\nu_{\mu}+\bar{\nu}_{\mu}}{\nu_{e}+\bar{\nu}_{e}}>2 \end{array}$ 

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### Multi-messenger Tomography of Earth

- Neutrino absorption tomography: Neutrino attenuation at energies greater than a few TeV. (• Placci, Alfredo and Zavattini, Emilio, 1973, https://cds.cern.ch/record/2258764 • L. Volkova and G. Zatsepin, Izvestiya Akademii Nauk SSSR, Seriya Fizicheskaya 38 (1974), no. 5 1060–1063. • Andrea Donini et. al. Nature Physics volume 15, pages 37–40 (2019))
- Neutrino oscillation tomography: While passing through Earth, neutrinos undergo charged-current coherent forward elastic scattering with ambient electrons and this results in the modification of neutrino oscillation patterns. This density-dependent matter effect can be used to reveal the internal structure of Earth. (L. Wolfenstein, Phys. Rev. D17 (1978) 2369)
- Neutrino diffraction tomography: The possibility of Earth tomography using the study of diffraction pattern produced by coherent neutrino scattering in crystalline matter inside Earth is technologically not feasible. (A. D. Fortes et. al. Using neutrino diffraction to study the Earth's core, Astronomy and Geophysics 47 (2006), no. 5 5.31–5.33.)

Probing Earth through neutrino absorption and oscillations is complimentary to seismic studies and gravitational measurement. This is the beginning of new era of **Multi-messenger tomography of Earth**.

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#### Neutrino Oscillations in Three-flavor Framework

$$u_{lpha} = \sum_{i} U_{lpha i} \nu_{i}$$

$$U = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta_{CP}} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta_{CP}} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

where,  $c_{ij} = \cos \theta_{ij}$  and  $s_{ij} = \sin \theta_{ij}$ .

$$P(\nu_{\alpha} \rightarrow \nu_{\beta}) = \left| U_{\beta 1} U_{\alpha 1}^{*} + U_{\beta 2} U_{\alpha 2}^{*} e^{-i2\alpha\Delta} + U_{\beta 3} U_{\alpha 3}^{*} e^{-i2\Delta} \right|^{2}$$
  
where,  $\Delta m_{ij}^{2} = m_{i}^{2} - m_{j}^{2}$ ,  $\alpha = \frac{\Delta m_{21}^{2}}{\Delta m_{31}^{2}}$  and  $\Delta = \frac{\Delta m_{31}^{2} L_{\nu}}{4E_{\nu}}$ 

• Normal Ordering (NO):  $(m_3 > m_2 > m_1)$  • Inverted Ordering (IO):  $(m_2 > m_1 > m_3)$ 

In this analysis, we use the three-flavor oscillation framework in the presence of matter (PREM profile) with the following values of the benchmark oscillation parameters.

$\sin^2 2\theta_{12}$	$\sin^2 \theta_{23}$	$\sin^2 2\theta_{13}$	$ \Delta m_{32}^2 $ (eV <sup>2</sup> )	$\Delta m_{21}^2$ (eV <sup>2</sup> )	$\delta_{\mathrm{CP}}$	Mass Ordering
0.855	0.5	0.0875	$2.46 imes10^{-3}$	$7.4 imes10^{-5}$	0	Normal (NO)

$$\Delta m_{\rm eff}^2 = \Delta m_{31}^2 - \Delta m_{21}^2 (\cos^2 \theta_{12} - \cos \delta_{\rm CP} \sin \theta_{13} \sin 2\theta_{12} \tan \theta_{23})$$

### Matter Effect in Neutrino Oscillations

The standard *W*-mediated matter potential  $V_{CC}$  experienced by neutrino/antineutrino during interaction with the ambient electrons in the matter can be expressed as

$$V_{CC} = \pm \sqrt{2} G_F N_e \approx \pm 7.6 \times Y_e \times 10^{-14} \left[ \frac{\rho}{\text{g/cm}^3} \right] \text{ eV}, \qquad (1)$$

where,

- $\rho$  denotes the matter density of various layers inside the Earth for a given profile
- $Y_e = N_e/(N_p + N_n)$  corresponds to the relative electron number density inside the matter. In the present analysis, we assume the Earth to be electrically neutral and isoscalar where  $N_n \approx N_p = N_e$ which results in  $Y_e = 0.5$ .
- The positive (negative) sign is for neutrino (antineutrino).
- Matter effect is significant for neutrino (antineutrino) for normal (inverted) ordering.

### Density Distribution of Various Profiles of Earth



Profiles	Layer boundaries (km)	Layer densities (g/cm <sup>3</sup> )		
PREM	25 layers	25 densities		
Core-mantle-crust	(0, 3480, 5701, 6371)	(11.37, 5, 3.3)		
Mantle-crust	(0, 5701, 6371)	(6.45, 3.3)		
Core-mantle	(0, 3480, 6371)	(11.37, 4.42)		
Uniform	(0, 6371)	(5.55)		

Note that while considering alternative profiles of Earth, we assume the radius and the mass of Earth to be invariant.

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### Effect of diff. Density Profiles on $P(\nu_{\mu} \rightarrow \nu_{\mu})$ Oscillograms



MSW resonance (L. Wolfenstein, Phys. Rev. D17 (1978) 2369): red patch around  $-0.8 < \cos \theta_{\nu} < -0.5$  and 6 GeV  $< E_{\nu} < 10$  GeV

Neutrino oscillation length resonance (Petcov, Phys. Lett. B 434 (1998) 321)/parametric resonance resonance (Akhmedov, Nucl. Phys. B538 (1999) 25): yellow patches around  $\cos \theta_{\nu} < -0.8$  and 3 GeV  $< E_{\nu} < 6$  GeV Anil Kumar et. al. JHEP 08 (2021) 139, arXiv: 2104.11740

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### Iron Calorimeter Detector (ICAL) at INO<sup>2</sup>

- ICAL@INO: 50 kton magnetized iron calorimeter detector at the proposed India-based Neutrino Observatory (INO)
- Location: Bodi West Hills, Theni District, Tamil Nadu, India
- Aim: To determine mass ordering and precision measurement of atmospheric oscillation parameters.
- Source: Atmospheric neutrinos and antineutrinos in the multi-GeV range of energies over a wide range of baselines.
- Uniqueness: Charge identification capability helps to distinguish  $\mu^-$  and  $\mu^+$  and hence,  $\nu_{\mu}$  and  $\bar{\nu}_{\mu}$
- Muon energy range: 1-25~GeV, Muon energy resolution:  $\sim 10\%$
- Baselines: 15 12000 km, Muon zenith angle resolution:  $\sim 1^\circ$

<sup>2</sup>Pramana - J Phys (2017) 88 : 79, arXiv:1505.07380



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### ICAL Design and Specfications



Resistive plate chamber (RPC) (active element) sandwiched between iron plates (passive element)

## Identifying Events for Neutrinos Passing through Different Layers of Earth

- Neutrino flux (Honda) at INO site
- 500 kt·yr exposure at ICAL
- Three-flavor neutrino oscillations in the presence of matter with the PREM profile
- Reconstructed muon events



Regions	$\cos  heta_{ u}$	$L_{ u}$ (km)	$\mu^-$ Events	$\mu^+$ Events
Crust-mantle-core	(-1.00, -0.84)	(10691, 12757)	331	146
Crust-mantle	(-0.84, -0.45)	(5721, 10691)	739	339
Crust	(-0.45, 0.00)	(437, 5721)	550	244
Downward	(0.00, 1.00)	(15, 437)	2994	1324
Total	(-1.00, 1.00)	(15, 12757)	4614	2053

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 11/15

# Distribution of Events for Neutrinos Passing through Different Layers of Earth



 $\bullet$  Neutrino flux at INO site  $\bullet$  500 kt·yr exposure at ICAL  $\bullet$  Three-flavor neutrino oscillations in the presence of matter with the PREM profile.

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## Effective Regions in $(E_{\mu}^{\text{rec}}, \cos \theta_{\mu}^{\text{rec}})$ Plane to Validate Earth's Core

MC Data: Core-mantle-crust 

 Theory: Mantle-crust 
 500 kt-yr exposure at ICAL 

 Systematic uncertainties are marginalized whereas oscillation parameters are kept fixed in theory



	Fixed-parameter $\Delta\chi^2$			
	NO	10		
Contribution from $\mu^-$	6.85	0.02		
Contribution from $\mu^+$	0.05	4.08		
Total	6.90	4.10		

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### Sensitivity to Validate Earth's Core with and without CID

• 500 kt·yr exposure at ICAL • Marginalization over systematic uncertainties and  $\sin^2 \theta_{23}$ : (0.36, 0.66),  $\Delta m_{\text{eff}}^2$ : (2.1, 2.6) ×10<sup>-3</sup> eV<sup>2</sup>, and mass ordering: (NO, IO)

		$\Delta\chi^2_{ICAL-profile}$					
MC Data	Theory	NO(†	true)	IO(true)			
		with CID	w/o CID	with CID	w/o CID		
PREM profile	Vacuum	5.52	3.52	4.09	1.67		
PREM profile	Mantle-crust	7.45	3.76	4.83	1.59		
PREM profile	Core-mantle	0.27	0.18	0.21	0.07		
PREM profile	Uniform	6.10	3.08	3.92	1.18		

 Anil Kumar et. al. JHEP 08 (2021) 139, arXiv: 2104.11740
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### Summary and Conclusion

- Atmospheric neutrinos can reveal the internal structure of Earth using matter effects in neutrino oscillations.
- ICAL can detect 331  $\mu^-$  and 146  $\mu^+$  core passing events in 10 years.
- The presence of Earth's core result in the neutrino oscillation length resonance or parametric resonance.
- The presence of Earth's core can be independently confirmed at ICAL with a median  $\Delta\chi^2$  of 7.45 (4.83) assuming normal (inverted) mass ordering

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### Thank you

### Effect of diff. Density Profiles on $P(\nu_e \rightarrow \nu_\mu)$ Oscillograms



 Anil Kumar et. al. JHEP 08 (2021) 139, arXiv: 2104.11740
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### Backup: Reconstructed Events at ICAL using various profiles of Earth

Profiles	Reconstructed $\mu^-$ events			Reconstructed $\mu^+$ events			
TTOMES	Upward	Downward	Total	Upward	Downward	Total	
PREM	1654	2960	4614	741	1313	2053	
Core-Mantle-Crust	1659	2960	4619	739	1313	2052	
Vacuum	1692	2960	4652	745	1313	2057	

 Anil Kumar et. al. JHEP 08 (2021) 139, arXiv: 2104.11740
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### Statistical Analysis

In this analysis, the  $\chi^2$  statistics is expected to give median sensitivity of the experiment in the frequentist approach.

$$\chi_{-}^{2} = \min_{\xi_{l}} \sum_{i=1}^{N_{E^{\prime}ec}} \sum_{j=1}^{N_{eee}} \sum_{k=1}^{N_{cos} \theta_{\mu}^{ee}} \left[ 2(N_{ijk}^{\text{theory}} - N_{ijk}^{\text{data}}) - 2N_{ijk}^{\text{data}} \ln\left(\frac{N_{ijk}^{\text{theory}}}{N_{ijk}^{\text{data}}}\right) \right] + \sum_{l=1}^{5} \xi_{l}^{2}$$

where,

$$\mathsf{V}^{ ext{theory}}_{ijk} = \mathsf{N}^{\mathsf{0}}_{ijk} \left( 1 + \sum_{l=1}^5 \pi^l_{ijk} \xi_l 
ight)$$

Similarly,  $\chi^2_+$  is defined for  $\mu^+$ 

$$\chi^2_{\rm ICAL} = \chi^2_- + \chi^2_+$$

 $\Delta \chi^2_{\text{ICAL-profile}} = \chi^2_{\text{ICAL}} \text{ (Mantle-Crust)} - \chi^2_{\text{ICAL}} \text{ (Core-Mantle-Crust)}$ 

• Marginalization over systematic uncertainties and  $\sin^2 \theta_{23}$ : (0.36, 0.66),  $\Delta m_{\text{eff}}^2$ : (2.1, 2.6) ×10<sup>-3</sup> eV<sup>2</sup>, and mass ordering: (NO, IO)

### Impact of Marginalization over Various Oscillation Parameters

- 500 kt·yr exposure at ICAL
- Marginalization over systematic uncertainties.
- Marginalization range for sin<sup>2</sup>  $\theta_{23}$ : (0.36, 0.66),  $|\Delta m_{eff}^2|$ : (2.1, 2.6) ×10<sup>-3</sup> eV<sup>2</sup>, and mass ordering: (NO, IO)

		$\Delta \chi^2_{ICAL-profile}$					
MC Data	Theory	Fixed	Marginalization over				
		parameter	$\sin^2 \theta_{23}$	$ \Delta m_{\rm eff}^2 $	$\pm  \Delta m_{ m eff}^2 $	All	
Core-mantle-crust	Mantle-crust	6.90	6.36	6.84	6.84	6.31	
Core-mantle-crust	Vacuum	6.80	6.44	5.16	4.94	4.65	
PREM	Mantle-crust	7.88	7.47	7.81	7.81	7.45	
PREM	Vacuum	7.71	7.28	6.10	5.89	5.52	

### Impact of Different True Choices of $\sin^2 \theta_{23}$

• 500 kt·yr exposure at ICAL • Marginalization over systematic uncertainties and oscillation parameters sin<sup>2</sup>  $\theta_{23}$ ,  $\Delta m_{\text{eff}}^2$ , and mass ordering.



 Anil Kumar et. al. JHEP 08 (2021) 139, arXiv: 2104.11740
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